One-Page Expert Ask: UC Rigidity Taxonomy

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Unique Continuation as a Universal Rigidity Principle: A Taxonomy of Success and Obstruction

1 The UC Engine (Abstract)

Six-Step UC Pipeline

1. **Carleman estimates:** Weighted L^2 control

2. **Three-cylinder/cone:** Quantitative propagation

3. **Doubling:** Iteration to explicit bounds

4. **Frequency:** Almgren-type frequency functionals

5. **Vanishing order:** Finite vanishing order from doubling

6. **Blow-up contradiction:** Finite order $\Rightarrow$ no singularity

2 The Form Class (Universal)

Form-Boundedness

$$\mathcal{A}_{V,r}(t) = \sup_{\|f\|_{H_0^1(B_r)}=1} \int_{B_r} |V||f|^2$$

Universal lemma: Energy $\Rightarrow \mathcal{A}_{V,r} \in L_t^2$
Covers: Vorticity (NS), curvature (YM, WM, HM), Schrödinger potentials, drift terms.

3 The Rigidity Theorem

UC Rigidity

If: Form-boundedness + UC-doubling + geometry supports backward propagation
Then: No finite-time singularities exist
Applies to: Elliptic, parabolic, hyperbolic (fixed geometry)
Examples: Schrödinger, NS, MHD, YM heat, HM heat, wave maps, YM wave

4 The Obstruction Theorem

> **Einstein Obstructions**
>
> **If:** Fully dynamical Lorentzian geometry (Einstein vacuum)
> **Then:** Six geometric obstructions prevent UC closure:
>
> 1. Trapped null geodesics
>
> 2. Non-coercive wave operator
>
> 3. Gauge instability
>
> 4. Weight degeneration
>
> 5. Backward uniqueness failure
>
> 6. Circularity of curvature control
>
> **Conclusion:** UC rigidity mechanism not structurally well-posed for fully dynamical geometries.

5 The Taxonomy

> **Classification**
>
> **Rigidity class $\mathcal{R}$:** UC + form-boundedness $\Rightarrow$ rigidity
> **Obstruction class $\mathcal{O}$:** Geometric obstructions prevent closure
> **Boundary:** Fixed geometry $\Rightarrow \mathcal{R}$, dynamical geometry $\Rightarrow \mathcal{O}$

6 Where to Attack if Wrong

> **Expert Questions**
>
> 1. **Is form-boundedness really universal?** Can you find a counterexample where form-boundedness holds but UC fails?
>
> 2. **Is the boundary correct?** Are there systems with "slowly varying" geometry that are in $\mathcal{R}$ but not $\mathcal{O}$?
>
> 3. **Are the Einstein obstructions complete?** Is there a way to overcome them with extra assumptions?
>
> 4. **Is the taxonomy complete?** Are there systems that don't fit into $\mathcal{R}$ or $\mathcal{O}$?